Conditions for anomalous acousto-optical diffraction by backward propagating acoustic waves

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(Received 13 July 2000; published 12 January 2001)

Experimental evidence is given of acoustic plate waves, whose group velocity is contradirected with respect to the phase velocity, through a determination of the acoustic wavelength dependence on frequency, in a limited range of frequencies. The dependence $d\Lambda/d\Omega>0$ between wavelength and frequency is experimentally verified, as the required condition for acousto-optical diffraction, where higher frequency components would scatter light into smaller diffraction angles.

DOI: 10.1103/PhysRevE.63.026604 PACS number(s): 43.35.+d

Few cases of acoustic wave propagation exist where dispersion relations cause the Poynting vector to reverse its direction with respect to the wave vector direction: guided acoustic waves in plates is one among a few examples, and backward propagating modes can be produced in a very limited number of cases, materials, and frequency ranges. Experimental evidence of backward propagating acoustic modes was given by few authors in the past, first by Meitzler [1] in thin wires and strips, then by Negishi [2] and Wolf *et al.* [3], who used a Schlieren technique to visualize waves propagating in a direction opposite to that of normal propagation.

In the present paper, the dispersion curve of the S_1 Lamb mode is experimentally tracked in an aluminum plate in the frequency range where backward propagation exists, through an acousto-optical determination of the acoustic wavelength. As properly described, this is experimental evidence of the condition of an anomalous acousto-optical diffraction effect, where high frequency acoustic components would diffract light into smaller angles than the lower components and spectral degeneracy would be produced, because optical beams with different frequency components would be scattered in the same directions.

Elastic energy propagates in thin plates as guided modes which expand freely into directions parallel to the plate surfaces, while satisfying transverse resonance boundary conditions. Polarization of the displacement vector may occur parallel to the surfaces (shear or Love modes) or on the sagittal plane normal to the surfaces (Lamb modes): in the first case, dispersion curves in isotropic materials do follow an equation, deduced by geometrical considerations,

$$K^2=(\Omega/v_s)^2-(n\pi/2b)^2, \qquad (1)$$

where $K=2\pi/\Lambda$ is the propagation wave vector, with Λ the acoustic wavelength, Ω is the angular frequency, v_s is the shear velocity in the medium of the plate, $2b$ is the plate thickness, and n is an integer $(n=1,2,\dots)$. In the case of Lamb modes, the coupling between shear and longitudinal components on the surface boundaries leads to the following dispersion relations, that can be solved numerically [4]:

$$\frac{\tan K_{ts}\,b}{\tan K_{tl}\,b}=-\left[\frac{4K^2K_{ts}K_{tl}}{(K_{ts}^2-K^2)^2}\right]^{\pm 1}, \qquad (2)$$

with plus and minus signs for symmetric and antisymmetric modes, respectively, and where $K_{ts}^2=(\Omega/v_s)^2-K^2$ and $K_{tl}^2=(\Omega/v_l)^2-K^2$ are the transverse wave numbers for shear and longitudinal waves, respectively. Some modes may show an anomalous behavior of $\Lambda(\Omega)$, within a specific range of frequencies, such that

$$d\Lambda/d\Omega>0, \qquad (3)$$

which causes a group of adjacent frequency waves to interfere constructively while progressing into directions opposite to the propagation direction of each single component wave, thus giving rise to the phenomenon of backward propagation, where the energy flow is contradirected with respect to the wave vector. This is the case for the S_1 symmetrical Lamb mode in aluminum plates in a frequency range below the $\Omega_0=\pi v_l/2b$ frequency that corresponds to $K=0$, as first shown by Tolstoy and Usdin [5]. Little experimental evidence was successively produced, and few uses of such phenomenon were proposed and exploited in the past [6,7]. It is shown here that this effect is a prerequisite for anomalous acousto-optical diffraction.

We suppose, indeed, that a plane light wave front $E(z,t)=E_0\exp\imath(kz+\omega t)$ of amplitude E_0, angular frequency ω, and wave number k impinges normally on the surface where an acoustic wave pulse $a(x,t)=\int a(\Omega)\cos[K(\Omega)x-\Omega t]d\Omega$ is propagating along a direction x; then it is reflected back with a spatial phase modulation as

$$\begin{aligned}E(x,z,t)&=E_0e^{\imath(kz-\omega t)}e^{2\imath a(x,t)k}\\&=E_0e^{\imath(kz-\omega t)}\Big[1+2\imath k\int a[\Omega(K)]\\&\quad\times\cos[K(\Omega)x-\Omega t]d\Omega+\cdots\Big].\end{aligned} \qquad (4)$$

In the usual case where $\lambda\ll\Lambda$, terms higher than first order can be dropped in the above expression and the Fourier transform of the optical field with respect to x becomes

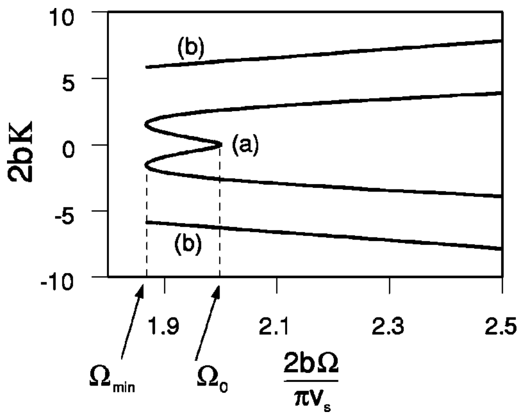

FIG. 1. Dispersion curve (a) of the S_1 mode in aluminum plates and asymptotic lines (b) $K=\pm\Omega/v_s$, around the region of backward propagation, in adimensional units.

$$E(\xi,z,t)\simeq E_0 e^{\imath(kz-\omega t)}\bigg[\delta(\xi)+2\imath k \times\int a[\Omega(K)]\{\delta[K(\Omega)-\xi]e^{-\imath\Omega t} + \delta[K(\Omega)+\xi]e^{+\imath\Omega t}\}\bigg]. \tag{5}$$

This is the undiffracted zeroth order, that corresponds to the $\delta(\xi)$ term, plus the first, right, and left diffracted orders, that correspond to the other terms, in directions

$$\theta_{\pm 1}=\tan^{-1}\left(\pm\frac{K}{k}\right), \tag{6}$$

each bearing a Doppler frequency shift with respect to the optical angular frequency of the undiffracted beam equal to $\Delta\omega_{\pm 1}=\pm\Omega$. The frequency spectrum of the acoustic wave is then reproduced in the spatial spectrum of the diffracted light, according to the one to one correspondence between angle and wave number given by

$$\theta_{\pm 1}=\tan^{-1}\left(\pm\frac{K(\Omega)}{k}\right)\simeq\pm\frac{K(\Omega)}{k}. \tag{7}$$

Within the angular spread $\Delta\theta$ corresponding to the frequency spread $\Delta\Omega$ of the acoustic pulse, the distribution of the optical frequency shift follows the law

$$\frac{\partial\theta}{\partial\Omega}=\frac{1}{k}\,\frac{\partial K(\Omega)}{\partial\Omega}=\pm\left(\frac{-2\pi}{k\Lambda^2}\,\frac{\partial\Lambda}{\partial\Omega}\right). \tag{8}$$

If $\partial\Lambda/\partial\Omega>0$, anomalous acusto-optical diffraction would occur, where higher acoustic frequency waves would diffract light into smaller angles than lower components. This is the case for backward propagating modes, which are possible in Lamb acoustic waves.

Figure 1 reports a slightly modified version of the well known branch of the S_1 mode dispersion curve in aluminum plates: the normalized propagation vector amplitude $2bK$ is plotted vs the normalized angular frequency $2b\Omega/\pi v_s$ in the symmetrical space of positive and negative values of K. The curve asymptotically follows the $K=\pm\Omega/v_s$ axes at high values of Ω, and is not defined in real space for Ω values below $\Omega_{min}<\Omega_0$. In the range $\Omega_{min}<\Omega<\Omega_0$, the group velocity $v_g=d\Omega/dK$ is contradirected with respect to the phase velocity $v_p=\Omega/K$.

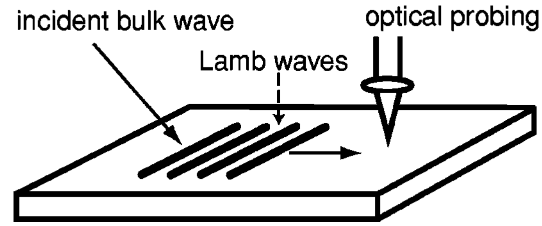

FIG. 2. Sketch of the experimental setup: the phase of acoustic plate waves (Lamb waves), generated by a mode conversion of the bulk wave, is optically measured.

As previously stated, the representation would be strictly followed by the acousto-optical diffraction spectrum of the wave where a one to one correspondence is present between the acoustic wave number K and the position x of the diffracted light spot. From the undiffracted light position $x=0$, that corresponds to a frequency Ω_0, up to one corresponding to the value Ω_{min}, anomalous diffraction exists, consisting of a decrease of the scattered light frequency at larger diffraction angles, or light spot positions. Maximum diffraction resolution occurs at $\Omega=\Omega_0$ and $\Omega=\Omega_{min}$, where $dK/d\Omega\rightarrow\infty$, while minimum diffraction corresponds to minimum group velocity or maximum negative value $(d\Omega/dK)_{max}$.

The experiments follow the dispersion law of the S_1 mode, as given in Fig. 1, through the $\Lambda(\Omega)$ curve determination, obtained by means of the phase lag evaluation of the acoustic wave at a given point and frequency with respect to a reference signal. Experiments (see Fig. 2) were performed on a $2b=2$ mm thick, carefully polished aluminum plate, where a wedge (variable angle) transducer generates sinusoidal Lamb waves by mode conversion, through a proper choice of both wedge angle and backward-wave region. The acoustic field is then probed by a laser beam impinging or-

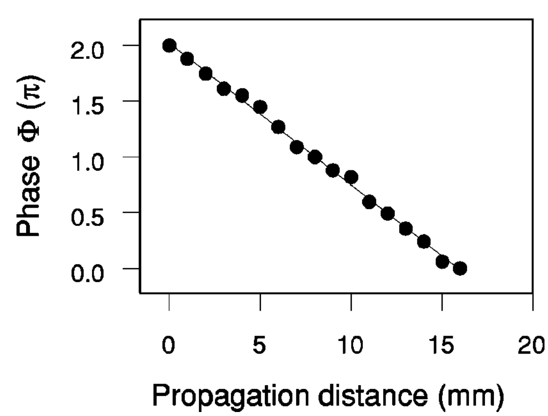

FIG. 3. Phase gain of an acoustic plate wave along the propagation distance.

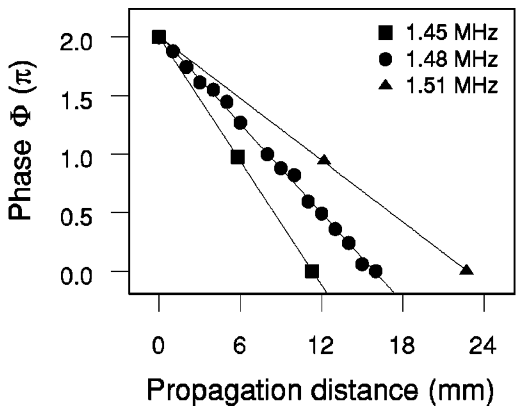

FIG. 4. Differential phase gain of acoustic plate waves at different frequencies: higher frequencies do propagate with larger wavelengths, thus causing a reversal of the energy propagation direction.

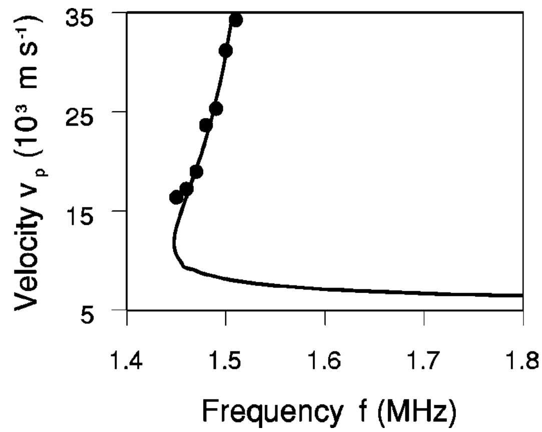

FIG. 5. Theoretical (line) and experimental data (dots) of the S_1 dispersion curve.

thogonally on the plate surface, that allows one, through an interferometric technique, to measure the surface vibration down to subnanometer amplitude values. It is then possible to detect the phase of the plate oscillation, by using laser light heterodyne detection with a Bragg frequency shifted reference beam. The resolution is noise limited and depends upon the surface optical quality; it typically lies between 0.25 and 2 nm.

Short pulses of acoustics waves 50 μs long were sent along the plate, and the central part of the pulse was monitored on an oscilloscope screen, as detected at any given point, thus allowing the phase relation to be measured with respect to the reference signal. Figure 3 shows the phase change Φ vs the laser beam position at increasing distances from the transducer along the acoustic beam path, at a frequency $f_1 = 1.48$ MHz: the wavelength (phase change equal to 2π) is 16 mm, corresponding to $K = 392.7$ m^{-1} and $v_p = 23680$ m/s. The negative slope indicates that the phase advances, as the probe is moved away from the transducer, as expected for backward wave propagation, since the energy is flowing away from the transducer, while the phase wave moves toward it. Figure 4 also reports results obtained at different frequencies $f_2 = 1.45$ MHz and $f_3 = 1.51$ MHz, which are 30 kHz below and above the reference frequency f_1. It can be seen that by increasing the frequency, the slope of the lines, and then the wave number *K*, decreases, and the wavelength increases, which is the condition for a backward wave to propagate.

Figure 5 shows experimental data for the phase velocity carried out at different frequencies in a range from 1.45 to 1.51 MHz at steps of 10 kHz, together with the theoretical dispersion curve of the Lamb mode S_1. There is a good agreement between experimental and theoretical data in most of the backward-wave region, down to $\Omega = \Omega_{min}$, where the group velocity reverses its sign, and the signal goes rapidly down to very low amplitudes, as already reported by Wolf *et al.* [3].

In conclusion, the determination of the acoustic wavelength of the S_1 mode in an isotropic plate of aluminum is done through optically probing the propagation surface, in the frequency range where backward propagation condition of the group velocity holds. Condition $d\Lambda/d\Omega > 0$ is experimentally verified, which gives rise to anomalous diffraction, where larger diffraction angles correspond to lower acoustic frequencies.

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